Resolution of Conflicting Objectives: A Utopia-Tracking Approach

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Multi-Objective Optimization

Conflicting Objectives Commonplace:

- Cost vs. Comfort
- Short-Term vs. Long-Term
- Stability vs. Robustness
- Expected Value vs. Risk
- Least-Squares vs. Prior
- Or Combinations (Energy vs. Comfort vs. Cost)

Multi-Objective Optimization

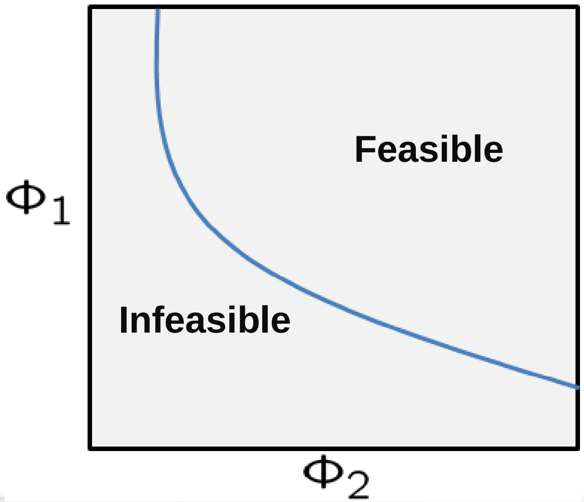
$$\min_x \Phi_1(x), \Phi_2(x), ..., \Phi_M(x)$$
$$\text{s.t. } g(x) \le 0$$

$\Phi_i(x), \quad i \in \mathcal{M}$ **Set of Objectives**

$g(x) \ge 0$ **Physical Model + Constraints**

- Conflicting: One Objective Cannot be Reduced without Increasing the Other(s)

Pareto Front



Multi-Objective Optimization

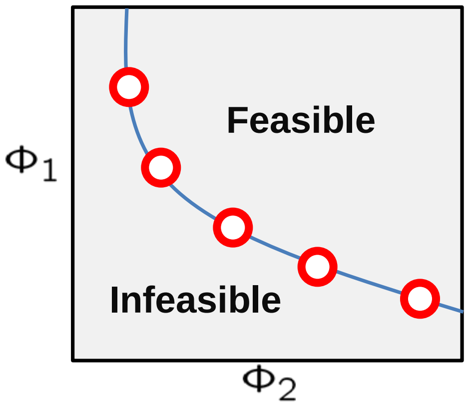
Typical Approach to Multi-Objective:

- **Choose Weights (Normally by Intuition or Biased Preference):**

$$\min_x \sum_{i=1}^{M} w_i \Phi_i(x) \qquad \sum_{i=1}^{M} w_i = 1.$$
$$\text{s.t. } g(x) \leq 0$$

Pareto Approach:

- **Try Combinations of Weights with** $\sum_{i=1}^{M} w_i = 1$ **To Construct Front and Pick <u>a</u> Solution**



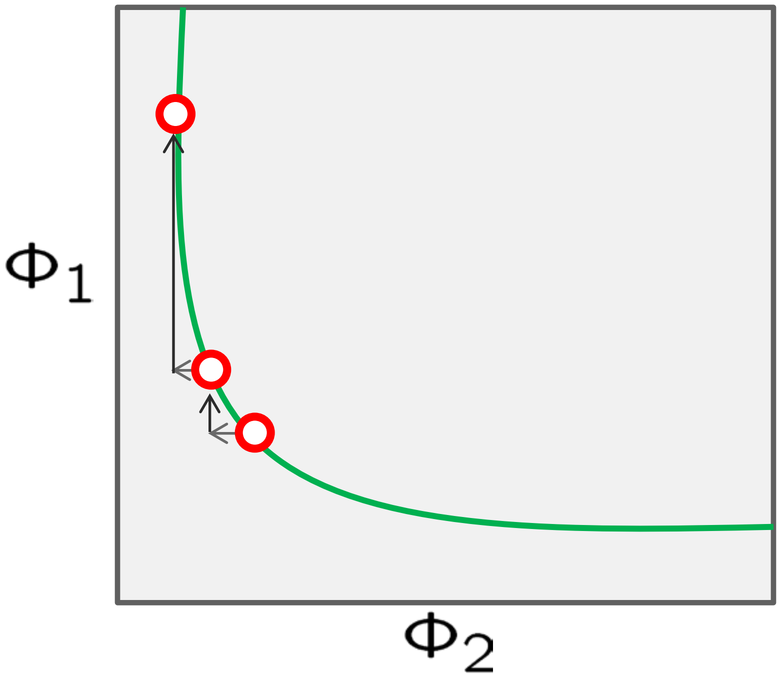
Issues with Pareto Approach:

- **Covering Domain Requires a Large Number of Points**
- **Front Might be Steep or Discontinuous (Solutions Might Not Exist for Trial Weights)**
- **How to Pick a Solution?**

Multi-Objective Optimization

Steepness of Pareto Front:

- **We Don't Know *a priori* how Sensitive is One Objective to Another**



- **Practitioner Can Place Weights in Region of Extreme Sensitivity**

- **Relaxing Objective by a Small Amount Leads to a Disproportionate Reduction in the Other(s)**

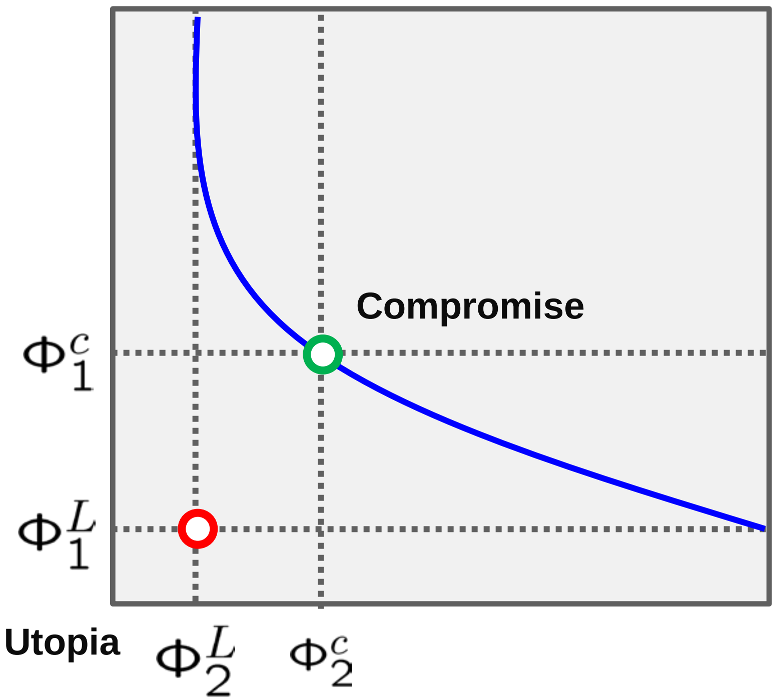
- **Number of Discretization Points Needed Increases with Steepness**

Utopia Tracking Approach

Utopia Point:

- **Point Where All Objectives are Individually Minimized (Ideal Performance)**

$$\min_x \Phi_i(x) \text{ s.t. } g(x) \leq 0 \longrightarrow \Phi_i^L, \quad i \in \mathcal{M}$$



Compromise Point:

- **Point of Consensus Among Objectives (Unbiased)**
- **Point Along Pareto Front that is the Closest to Utopia Point (In Some Norm)**
- **Coordinates Can be Obtained by Solving:**

$$\min_x \|\Phi(x) - \Phi^L\|_p \text{ s.t. } g(x) \leq 0 \longrightarrow \Phi_i^c, \quad i \in \mathcal{M}$$

Utopia Tracking Approach

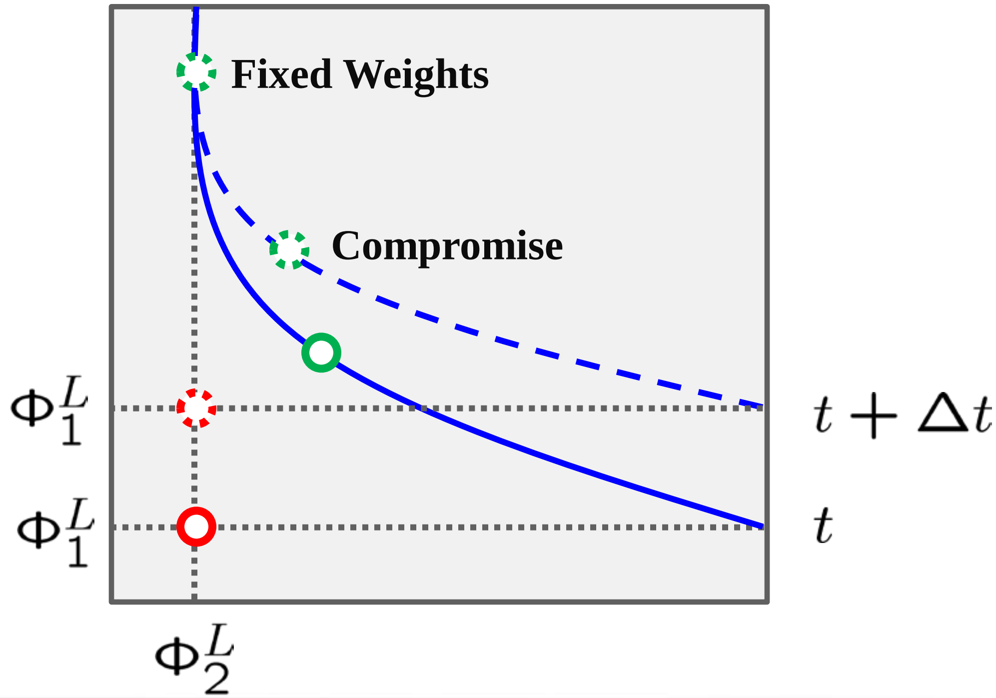
Benefits:

- **<u>Pareto Front Is Not Required</u>**
  - **Only Needs To Solve $M+1$ Problems**
  - **Example: Case with 2 Objectives Needs 3 Problems**
- **Scalable to Multiple Objectives**
- **Automatically Finds Weights**
- **Applicable To <u>Any</u> Type of Problem (Continuous, Discrete, Differential Equations)**

Utopia Tracking Approach

Benefits for Real-Time Control/Energy Management:

- **Pareto Front Changes in Time**
    - **e.g.; Changes with <u>Data</u> (e.g., Weather, Prices, Occupancy)**
- **Cannot Afford to Compute Pareto Front at Each Point in Time**
- **Using Time-Invariant Weights (Current Practice) : Bad Idea**



Implementation of Utopia Approach

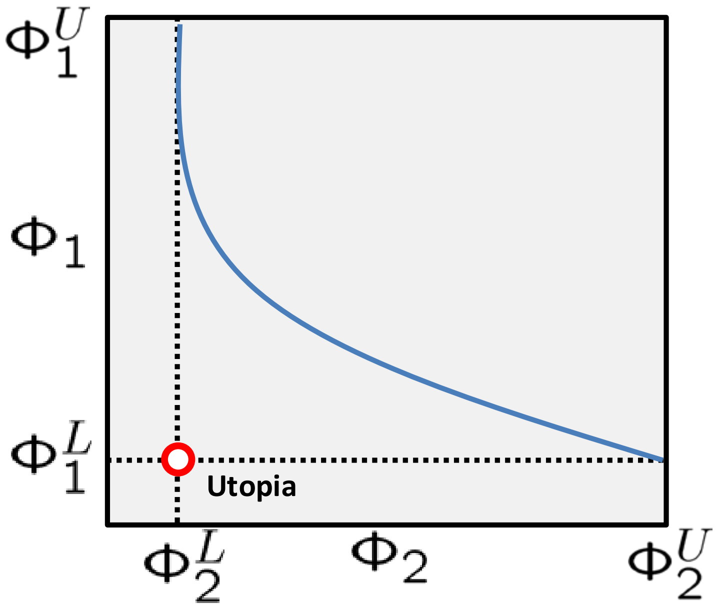
Scaling:

- **Objective Functions Can have Drastically Different Values**

- **Apply Normalization:**

$$\bar{\Phi}_i(x) \leftarrow \frac{\Phi_i(x) - \Phi_i^L}{\Phi_i^U - \Phi_i^L}, \qquad \bar{\Phi}_i(x) \in [0,1]$$

- **Upper Bounds Indirect Outcome of Utopia Subproblems** $\min_x \Phi_i(x)$ s.t. $g(x) \leq 0$



- **Compromise Problem Becomes**

$$\min_x \|\bar{\Phi}(x)\|_p \text{ s.t. } g(x) \leq 0$$

Implementation of Utopia Approach

- **L-Infinity Norm (Minimize the Maximum Distance Among Objectives)**

$$\|\bar{\Phi}(x)\|_\infty = \max_i \{|\bar{\Phi}_i(x)|\}$$

- **Leads to Nested Optimization Problem (Extremely Hard or Impossible to Solve)**

$$\min_x \|\bar{\Phi}(x)\|_\infty \text{ s.t. } g(x) \leq 0$$

- **Reformulate as:**

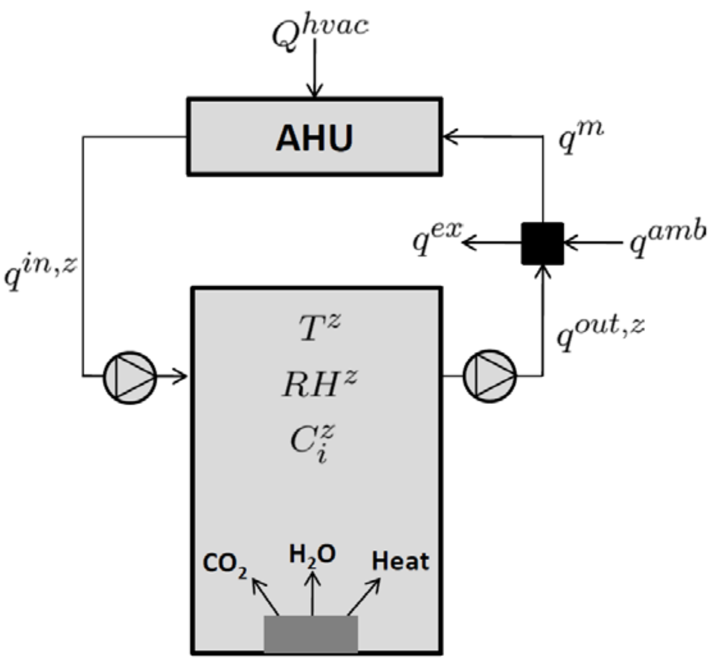
$$\begin{aligned} \min_x \quad & \eta \\ \text{s.t. } & g(x) \leq 0 \\ & \bar{\Phi}_i(x) \leq \eta, i \in \mathcal{M} \end{aligned}$$

$$\eta^* = \max_i \{|\bar{\Phi}_i(x)|\}$$

- **Because we know that, by construction,** $|\bar{\Phi}_i(x)| = \bar{\Phi}_i(x) \geq 0, \ i \in \mathcal{M}$

Numerical Study

Multi-Objective Optimal Control



- **Obj1: Minimize Energy**

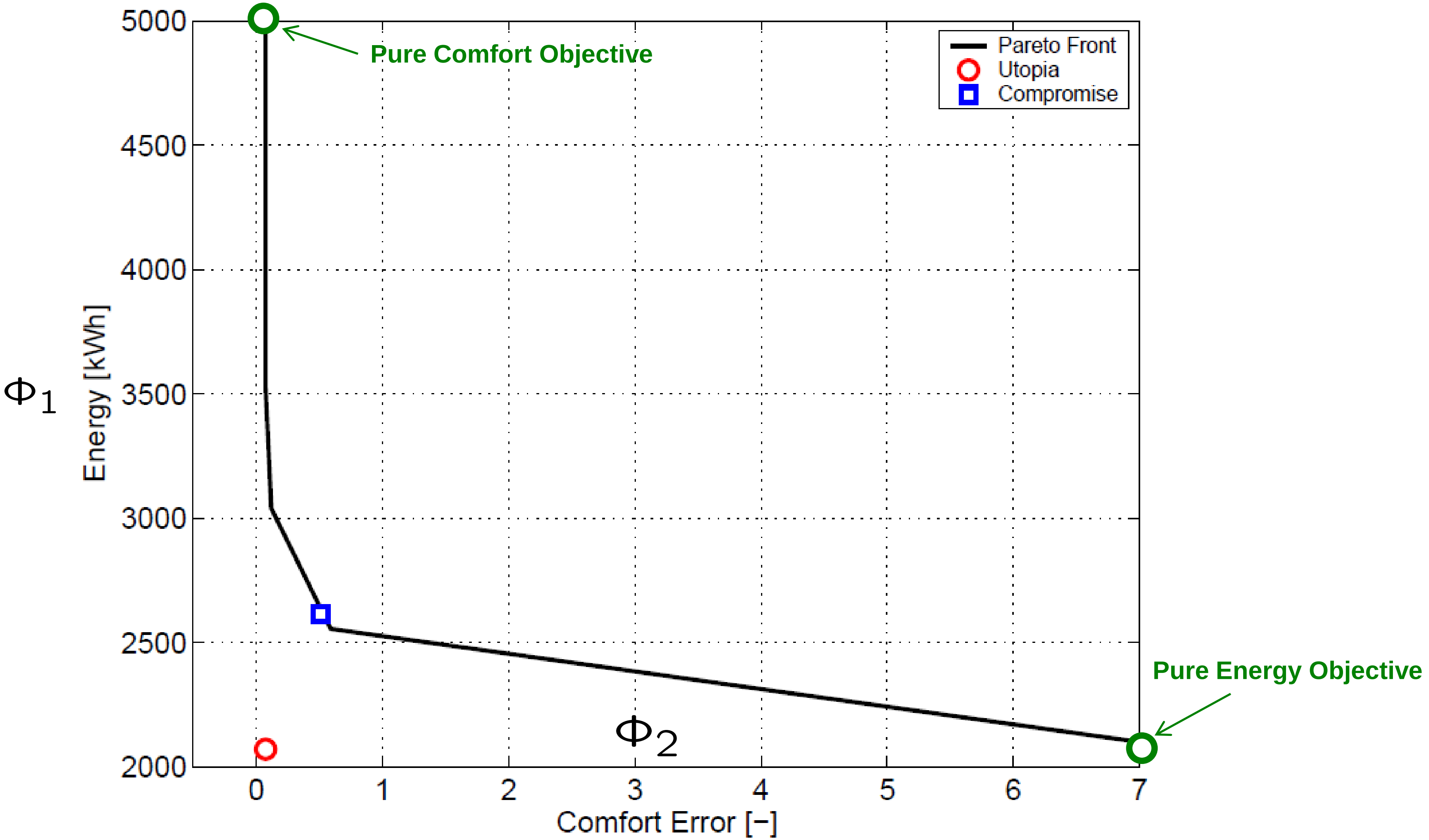
$$\Phi_1 = \int_0^T Q_{hvac}(\tau) d\tau$$

- **Obj2: Maximize Comfort**

$$\Phi_2 = \int_0^T \left(\|T(\tau) - T^{comfort} + \|RH(\tau) - RH^{comfort}\| \right) d\tau$$

- **Comfort Conditions Chosen for PPD = 1%.**
- **Model Description Available in Conference Paper (Energy and Mass Balances)**
- **All Problems Modeled in AMPL and Solved with IPOPT (See Friday Talk)**

Pareto Front : Comfort vs. Energy



- **Slight Relaxation of Comfort Leads to <u>Disproportionate</u> Reductions in Energy**
- **Computing Pareto Front Required <u>10 Hours of Computation</u> (1,000 Points)**
- **Computing Compromise Point Required <u>2 Minutes</u> (3 Points)**

Conclusions and Open Questions

- **Pareto Front Not Needed to Make Decisions**

- **Focus on Limiting Behavior (Utopia Point) and Try to Get Close to It**

- **Proposed Approach is Scalable to Multiple Objectives and Different Problem Classes**

- **Applications:**
    - **Design**
    - **Retrofit Analysis**
    - **Control/Energy Management**
    - **Estimation**

- **Open Questions**
    - **Formulations Under Uncertainty (What if Data is Uncertain?)**
    - **Rigorous Analysis of Energy vs. Comfort Trade-Off (Where is Compromise?)**
    - **Real-Time Multi-Objective Control (Stability of Compromise Solution?)**

Financial Support

DOE Building Technologies Program

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